Identifiability Matters: Revealing the Hidden Recoverable Condition in Unbiased Learning to Rank

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1. Background

❶ **Traditional Learning to Rank (LTR)**: Given a dataset $\{(\boldsymbol{x}_i, y_i)\}_{i=1}^N$

- $\boldsymbol{x}_i$: ranking feature; y_i: relevance label.

❷ **Biased Learning to Rank with clicks**: Given a dataset $\{(\boldsymbol{x}_i, c_i)\}_{i=1}^N$

- c_i: user clicks — easy to obtain at scale, but *bias* from the label y_i.
- **Example**: low-ranked documents are less likely to be clicked (position bias).
- **Reason**: Some bias factors (e.g., position, denoted as $\boldsymbol{t}$) affect user observation.
- **Solution**: Using *examination hypothesis* (Joachims et al., 2017) to decompose the biased clicks into relevance probability and observation probability:

$$\underbrace{c(\boldsymbol{x},\boldsymbol{t})}_{\text{click probability}} = \underbrace{r(\boldsymbol{x})}_{\text{relevance probability}} \cdot \underbrace{o(\boldsymbol{t})}_{\text{observation probability}}$$

❸ **Unbiased Learning to Rank (ULTR)**: Given a dataset $\{(\boldsymbol{x}_i, \boldsymbol{t}_i, c_i)\}_{i=1}^N$ with bias factors $\boldsymbol{t}_i$

- Train two models, ranking model $r'(\cdot)$ and observation models $o'(\cdot)$, to fit clicks based on examination hypothesis:

$$\mathcal{L} = \sum_{i=1}^{N} \text{loss}(r'(\boldsymbol{x}_i)\cdot o'(\boldsymbol{t}_i), c_i) \xrightarrow{\text{After training}} r'(\boldsymbol{x})\cdot o'(\boldsymbol{t}) = c(\boldsymbol{x},\boldsymbol{t})$$

Problem: After training, can we recover the true relevance r through the model r'?

$$r(\boldsymbol{x})\cdot o(\boldsymbol{t}) = r'(\boldsymbol{x})\cdot o'(\boldsymbol{t}) \xrightarrow{\text{can we guarantee?}} r \text{ and } r' \text{ imply the same rankings.}$$

★ **Our contribution:** find the identifiable condition to recover the true relevance, which can be converted to a **graph connectivity test**. This builds a solid ground for ULTR.

2. Main Result: Identifiability Theory

Definition 1 (Identifiability). *We say that the relevance model is identifiable, if*

$$r(\boldsymbol{x})\cdot o(\boldsymbol{t}) = r'(\boldsymbol{x})\cdot o'(\boldsymbol{t}) \implies r(\boldsymbol{x})/r'(\boldsymbol{x}) = \text{constant}.$$

Identifiability up to a scaling transformation is sufficient for pairwise ranking objectives.

Our following main result establishes that **identifiability is intrinsically linked to the dataset** $\mathcal{D} = \{(\boldsymbol{x}_i, \boldsymbol{t}_i)\}_{i=1}^N$, which can be readily mined:

Theorem 1 (Equivalent condition of identifiability). *The relevance model is identifiable, if and only if an undirected graph $G = (V, E)$ is connected, where V is a node set and E is an edge set, defined as:*

$$V = \{v_1, v_2, \cdots, v_{|\mathcal{T}|}\},$$
$$E = \{(v_s, v_t) \mid \exists \boldsymbol{x} \in \mathcal{X}, \text{ s.t. } (\boldsymbol{x}, \boldsymbol{s}) \in \mathcal{D} \wedge (\boldsymbol{x}, \boldsymbol{t}) \in \mathcal{D}\},$$

where $\mathcal{T}$ is the range of bias factors. We refer to this graph as **identifiability graph** (IG).

Key insights:

- Identifiability of the ranking models only depends on the dataset.
- Smaller datasets, more features, or more bias factors lead to unidentifiability easily.
- Unidentifiability exists in a real-world TianGong-ST dataset (Chen et al., 2019).

3. An Example for applying Theorem 1

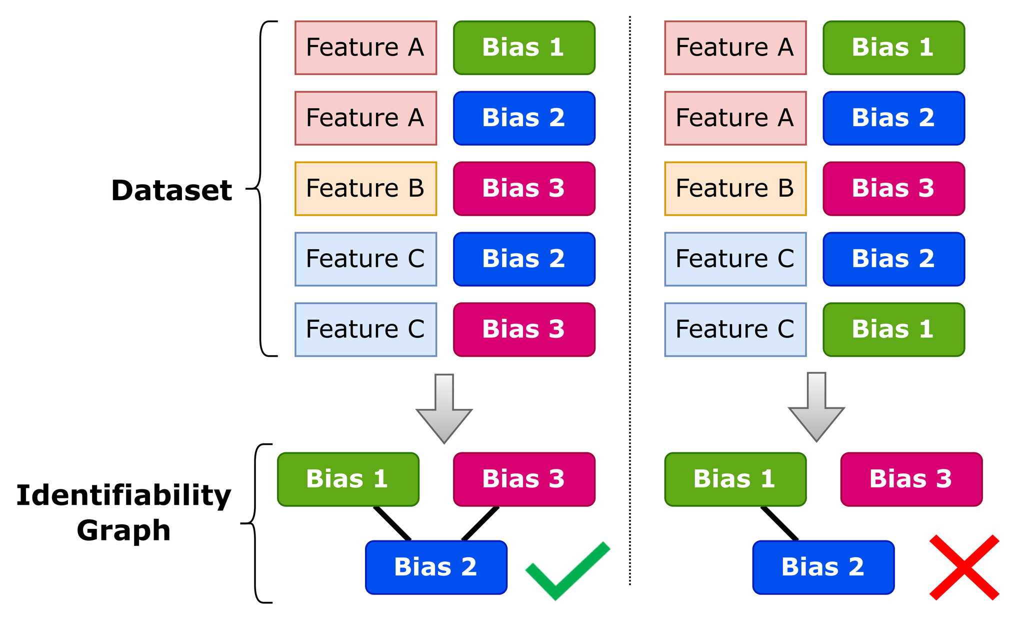


Figure 1. Examples for identifiable and unidentifiable case.

Theorem 1 shows that:

identifiability ↔ connectivity

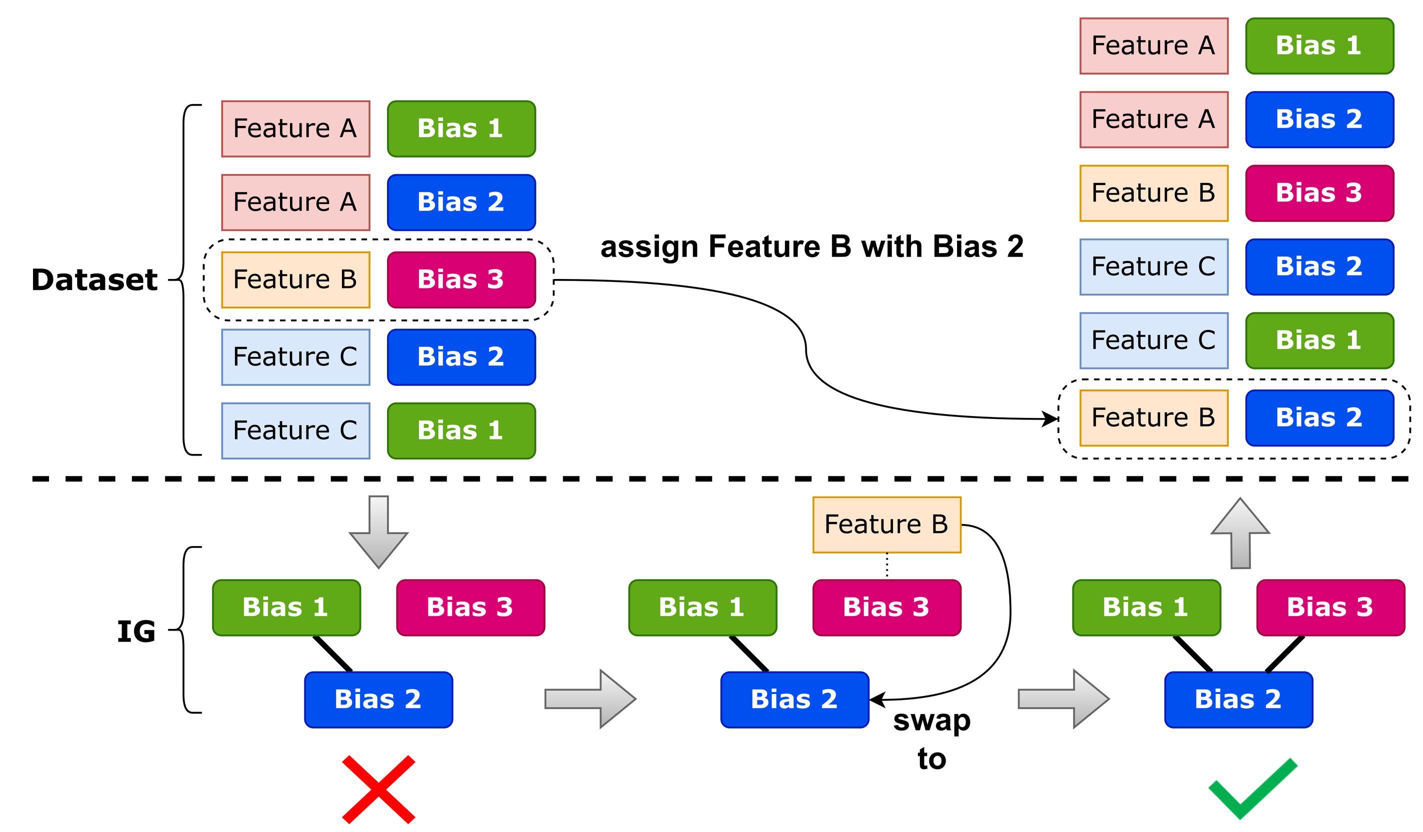
Figure 1 (Left): Bias 1 and Bias 2 are connected through Feature A. Bias 2 and Bias 3 are connected through Feature C. As a result, the graph is *connected* and the ranking model is *identifiable*.

Figure 1 (Right): Bias 3 remains isolated, leading to *unidentifiability*.

4. Dealing with Unidentifiable Dataset

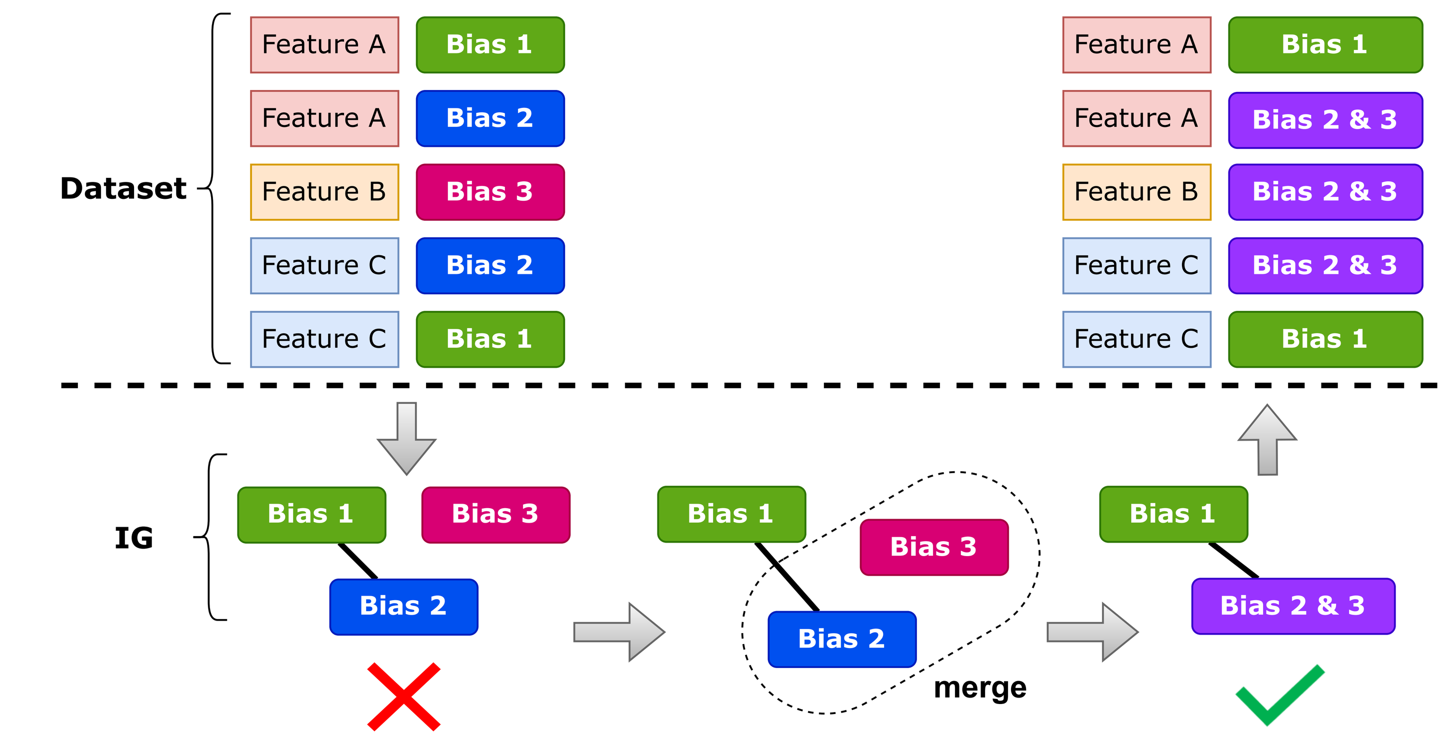
For an unidentifiable dataset, the IG is disconnected. Therefore, we propose two methods applied to the dataset to establish connectivity within the corresponding IG.

❶ **Node intervention**: Augment datasets by swapping some pairs of documents between different bias factors.



- Compared to traditional intervention strategies (Joachims et al., 2017; Yue et al., 2010), node intervention requires much fewer interventions.
- It still requires additional data from online experiments.

❷ **Node merging**: Merging nodes from different connected components and treating them as a single node, to connect the IG.



- Node merging performs on the offline dataset, without the need for additional data.
- However, merging bias factors brings additional approximation error.
- We prove that: **such approximation error is bounded by the diameter of a minimum spanning tree related to this process.**

5. Experiment

Table 1. Performance of different methods on the simulated unidentifiable dataset.

Method	MCC ↑	nDCG@1 ↑	nDCG@3 ↑	nDCG@5 ↑	nDCG@10 ↑	Click MSE
No debias	$0.521_{\pm.000}$	$0.711_{\pm.000}$	$0.625_{\pm.000}$	$0.665_{\pm.000}$	$0.820_{\pm.000}$	2×10^{-5}
DLA	$0.707_{\pm.105}$	$0.836_{\pm.061}$	$0.742_{\pm.091}$	$0.789_{\pm.070}$	$0.886_{\pm.040}$	$<10^{-8}$
+ Node intervention	$\mathbf{1.000}_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$<10^{-8}$
+ Node merging	$0.975_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$<10^{-8}$
Regression-EM	$0.580_{\pm.117}$	$0.786_{\pm.035}$	$0.677_{\pm.063}$	$0.752_{\pm.044}$	$0.857_{\pm.027}$	$<10^{-8}$
+ Node intervention	$\mathbf{0.980}_{\pm.023}$	$0.999_{\pm.001}$	$0.995_{\pm.010}$	$0.989_{\pm.023}$	$0.997_{\pm.006}$	$<10^{-7}$
+ Node merging	$0.975_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$<10^{-8}$
Two-Tower	$0.830_{\pm.050}$	$0.883_{\pm.034}$	$0.832_{\pm.054}$	$0.857_{\pm.045}$	$0.925_{\pm.022}$	$<10^{-8}$
+ Node intervention	$\mathbf{1.000}_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$<10^{-8}$
+ Node merging	$0.975_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$\mathbf{1.000}_{\pm.000}$	$<10^{-8}$

- Various ULTR algorithms yield disparate suboptimal performances when unidentifiable.
- With identifiability established through our approaches, they all converge to a common case that accurately recovers relevance.

6. Conclusion

- We propose the concept of identifiability and frame it as a graph connectivity test.
- We propose model-agnostic methods to handle the unidentifiable cases by restoring graph connectivity.

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